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Efficient analysis of 2D cloaking with 3D inclusions and of related bandwidth limitation METAMATERIALS 2013

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***Abstract* – 2D structures made of 3D metamaterial inclusions often exhibit a sectoral symmetry. This enables a vastly accelerated Method-of-Moment analysis of such structures. Further acceleration is offered by the multipole decomposition of the linear infinite-array Green's function. Such methods allow the fast frequency-domain response of cloaking metamaterials and in particular of their maximum bandwidth, which is shrinking for larger objects in terms of wavelengths, due to causality limitations.**

I. INTRODUCTION

Since the initial demonstration by Schurig et al. [1] in 2006, cloaking has attracted a lot of attention. This paper refers to 2D cloaking achieved with 3D inclusions, where the cloaking device comprises an inner conducting cylinder and has either axial symmetry, or -more generally- is a discrete body of revolution. It is made invisible to a plane wave whose direction of propagation is perpendicular to the object's axis and has a polarization that is either parallel or perpendicular to the structure axis.

The objective of the paper is two-fold. First, we show how such structures can be efficiently analyzed with the help of the Method of Moments. This allows the fast analysis of the perturbation of fields just behind the structure and of its total scattering cross-section, as initiated in [2] and [3]. Such analysis leads to the second objective, related to the bandwidth of the cloaking operation. Indeed, wave packets need to travel around the object to be cloaked and must suffer some delay, as compared to wave packets that would have simply propagated straight in free space. Such considerations led to a rule of thumb proposed in [4], where fields just behind the cloak were analyzed. Additional insights will be provided here by establishing a link with the total cross-section.

Section II details the numerical analysis of 2D cloaking with 3D inclusions and Section III treats the bandwidth limitations and Section IV shows numerical results. Conclusions are provided in Section V.

II. FAST NUMERICAL ANALYSIS

Cloaking structures should make objects invisible from any perspective. If we limit ourselves to any direction parallel to a plane (2D cloaking), structures with cylindrical symmetry are expected. Metamaterials have been shown to provide the necessary variations in permittivity and permeability in order to realize, at least approximately, the needed channeling of light. Those metamaterials are generally made of structured materials, comprising inclusions like split-ring resonators, for instance. With such structuration, the structures used for 2D cloaking are no longer exactly cylindrically symmetric but may correspond to discrete bodies of revolution, i.e. objects made of a number of identical sectors, while they are infinitely periodic in the axial direction.

The fast analysis of such cloaking structures has been analyzed in [2], with an extension to multipole analysis in [3]. The use of the periodic Green's function allows the implicit inclusion of the periodicity along the structure axis. The fast study of discrete bodies of revolution with the Method of Moments (MoM) essentially proceeds as follows. First, the MoM impedance matrix has a block-circulant structure, which obviously comes from the fact that the matrix can be partitioned into blocks that only depend on the angle separating two given sectors. Then, if there are N sectors, the excitation can be decomposed into N excitations that have sectoral symmetry, within a given phase shift between sectors. The solution of the periodic problem has a complexity proportional to the number of unknowns on one given sector. This leads to a savings of order N in terms of matrix filling and of order N^2 in terms of solution time. Examples of this computation strategy have been shown in [2],[3] for cloaks made of metallic cones [5] and of a combination of plates with split-ring resonators [6].

A further acceleration is offered by an original fast multipole approach, as initiated in [3]. Here, the periodic Green's function is decomposed into a number of cylindrical waves and the addition theorem for Bessel functions can be exploited for each of them. Hence, for each Floquet mode, the interactions between subdomains can be diagonalized, as a spectral summation on all directions over the unit circle, via the patterns of the subdomains and the multipole translation function. Interestingly, although the structure is three-dimensional, the integration can be limited to a circle. The effect of the third dimension is accounted for through the infinite summation over Floquet modes. It is also interesting to notice that, in the present application, this summation is extremely rapidly converging. Indeed, the structured metamaterial is generally made of small inclusions, such that the periodicity along the axis is much smaller than the wavelength. Besides, multipoles expansions are used only for lateral (i.e. orthogonal to axis) distance larger than about half wavelength. For such distances, at most, only the -1 to +1 Floquet modes need to be included. Hence, in practice, the infinite summation is limited to three terms.

III. Bandwidth Limitation

As discussed in the literature, cloaking suffers from a bandwidth limitation that stems from a causality issue (signals arriving "late"), which has been treated as follows in [4]. If cloaking is effective over a certain bandwidth, the propagation of a pulse of width equal to the inverse of the bandwidth should be essentially unaffected by the cloak and hidden object. The corollary question is that of the criterion to be used for the quality of cloaking: here, as a simple rule, we consider that the pulse can only suffer a delay that is small compared to its own duration. This approach might be limited to objects larger than the wavelength λ, or of the same order. This line of reasoning rapidly leads to the rule proposed in [4]: the relative bandwidth should be smaller than $\lambda/\Delta r$ $1/Q$, where Δr is the extra path followed by the pulse around the object to be cloaked, while Q is a quality factor, accounting for the acceptable level of cloaking degradation away from the central frequency. For relatively thin cloaking metamaterials, Δr is simply proportional to the size of the object to be cloaked. In other words, the relative cloaking bandwidth is inversely proportional to the size of the object expressed in wavelengths.

The rule provided above, stemming from pulse delay considerations, essentially refers to the quality of fields just behind the cloak. Another important type of criterion refers to the quality of far fields, for instance in the form of total scattering cross-section. In an attempt [7] in that direction, the fields around the object are modeled with several optimistic assumptions, enforcing however a limitation on group velocity around the cloak. Surface equivalence is then used to estimate the total scattering cross-section. For a given reduction of cross-section, the relative bandwidth is then also found proportional to the inverse size of the object, which allows the establishment of a link between the quality factor Q referred to above and the reduction of total cross-section.

IV. Simulation examples

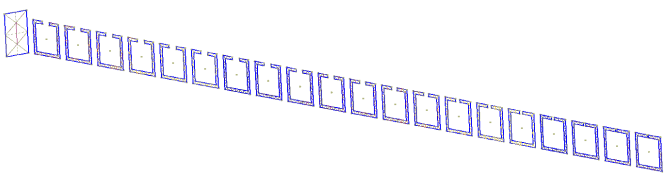

Fig. 1. Sector of the 144-sector cloaking structure proposed in [8].

Figure 1 corresponds to one sector of the cloaking structured proposed in [8], made of 144 sectors. In view of the large number of sectors, our method offers very large time savings. To the left of the figure, one can see a portion of the cylindrical conducting core, while the other elements correspond to split-ring resonators, each with a slightly different geometry (smaller gap toward the outside). Simulation results obtained for an incident plane wave are shown in Fig. 2. While an important shadow is visible when only the cylindrical core is present (left), a clearly reduced scattering is observed in the presence of the cloak (right). The bandwidth analysis for this cloak still needs to be carried out.

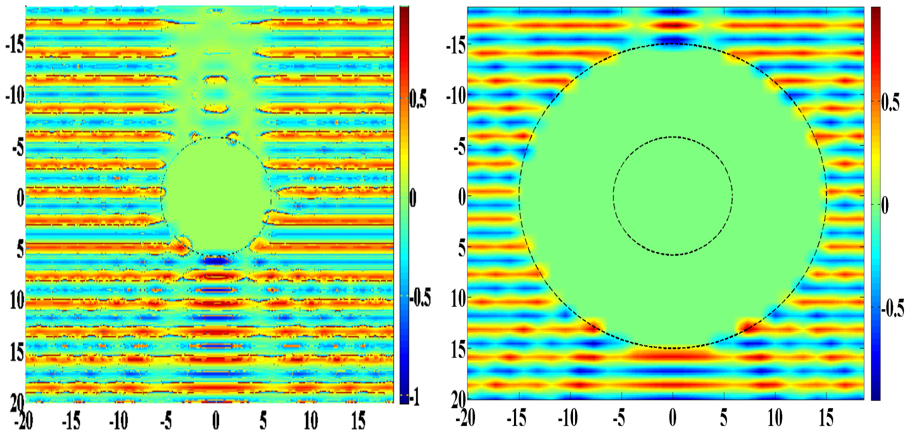

Fig. 2. Scattering by the cloak proposed in [8] using the proposed fast integral-equation approach. Left: conducting cylinder alone. Right: conducting cylinder and cloaking structure.

VI. Conclusion

We presented a fast integral-equation method for the analysis of 2D cloaks made of three-dimensional elements; the method is accelerated exploiting the sectoral symmetry of most of such cloaks as well as an original multipole formulation based on the infinite-array Green's function. This allows the fast analysis of the residual near-field perturbation by the cloak and of its total scattering cross-section, both of which correspond to reference quantities for the estimation of bandwidth limits of cloaks.

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